Class XII Session 2025-26
Subject - Applied Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

Read the following instructions very carefully and strictly follow them:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are case study-based questions carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section C, 2 questions in Section D and one sub-part each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. The value of x for which the points (2, -1), (-3, 4) and (x, 5) are collinear, is: **[1]**

a) 4 b) -2

c) 2 d) -4

2. For a student's t-test, the test statistic t is given by: **[1]**

a) $t = \frac{\bar{x}}{s}$ b) $t = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n-1}}}$

c) $t = \bar{x} - \mu$ d) $t = \frac{\bar{x}-\mu}{s}$

3. A customer deposits ₹ 2400 at 5% p.a. compounded half yearly. At the end of the year the interest earned will be **[1]**

a) ₹ 123 b) ₹ 121.44

c) ₹ 122 d) ₹ 120

4. If the objective function for an L.P.P. is Z = 5x + 7y and the corner points of the bounded feasible region are (0, 0), (7, 0), (3,4) and (0, 2), then the maximum value of Z occurs at **[1]**

a) (0, 0) b) (0, 2)

c) (7, 0) d) (3, 4)

[1]

5. The value of $\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix}$ is

a) a + b + c

b) abc

c) 3 + abc

d) 1 + a + b + c

6. If X is a Poisson variable such that P(X = 1) = 2P(X = 2), then P(X = 0) is **[1]**

a) e

b) $\frac{1}{e}$

c) 1

d) e^2

7. If X is a random-variable with probability distribution as given below: **[1]**

$X = x_i$	0	1	2	3
$P(X = x_i)$	k	3k	3k	k

The value of k and its variance are:

a) $\frac{1}{8}, \frac{24}{27}$

b) $\frac{1}{8}, \frac{22}{27}$

c) $\frac{1}{8}, \frac{3}{4}$

d) $\frac{1}{8}, \frac{23}{27}$

8. Integrating factor of the differential equation $x\frac{dy}{dx} + y = 3x^2$ is **[1]**

a) x

b) $\frac{1}{x}$

c) log x

d) $\sqrt{x}$

9. A man can row a boat in still water at 15 km/hr and speed of water current is 5 km/hr. The distance covered by the boat downstream in 24 minutes is **[1]**

a) 8 km

b) 16 km

c) 4 km

d) 6 km

10. A matrix has 18 elements, the possible number of orders of a matrix are: **[1]**

a) 6

b) 5

c) 4

d) 3

11. (49 + 57) (mod 50) is **[1]**

a) 5

b) 6

c) 7

d) 4

12. If 0 < x < 1, which of the following is greatest? **[1]**

a) $\frac{1}{x}$

b) x

c) $\frac{1}{x^2}$

d) x^2

13. In a game of 100 points, A can give B 10 points and C 18 points. Then, B can give C: **[1]**

a) 45 : 41

b) 35 : 12

c) 35 : 41

d) 55 : 25

14. The feasible solution of an LPP belongs to: **[1]**

a) Second quadrant

b) Only first quadrant

c) First and second quadrants
d) First and third quadrants

15. The region represented by the inequation system x, y $\geq$ 0, y $\leq$ 6, x + y $\leq$ 3 is **[1]**

a) unbounded in first quadrant
b) bounded in second quadrant
c) bounded in first quadrant
d) unbounded in first and second quadrants

16. A sample of 50 bulbs is taken at random. Out of 50 we found 15 bulbs are of Bajaj, 17 are of Surya and 18 are of Crompton. What is the point estimate of population proportion of Surya? **[1]**

a) 0.34
b) 0.3
c) 0.36
d) 0.4

17. If $\int \frac{2^{1/x}}{x^2} dx$ = k $2^{1/x}$ + C, then k is equal to **[1]**

a) $-\log_e 2$
b) -1
c) $-\frac{1}{\log_e 2}$
d) $\frac{1}{2}$

18. For predicting the straight line trend in the sales of scooters (in thousands) on the basis of 6 consecutive years data, the company makes use of 4-year moving averages method. If the sales of scooters for respective years are a, b, c, d, e and f respectively, then which of the following average will not be computed? **[1]**

a) $\frac{c+d+e+f}{4}$
b) $\frac{a+c+d+e}{4}$
c) $\frac{b+c+d+e}{4}$
d) $\frac{a+b+c+d}{4}$

19. **Assertion (A):** If A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, then A + B = $\begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$. **[1]**

Reason (R): Two different matrices can be added only if they are of same order.

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

20. **Assertion (A):** The equation of all lines having slope 0. which are tangents to the curve y = $\frac{1}{x^2-2x+3}$, is y = $\frac{1}{2}$. **[1]**

Reason (R): The point at which tangent to the given curve having slope 0, is (1, $\frac{1}{2}$).

a) Both A and R are true and R is the correct explanation of A.
b) Both A and R are true but R is not the correct explanation of A.
c) A is true but R is false.
d) A is false but R is true.

Section B

21. Find the effective rate of return equivalent to a nominal rate of 6% per annum compounded **[2]**

i. quarterly
ii. continuously. (Given $(1.015)^4$ = 1.06136 and $e^{0.06}$ = 1.06183)

OR

A person amortizes a loan of ₹150000 for a new home by obtaining a 10 year mortgage at the rate of 12% compounded monthly. Find

i. EMI.
ii. Total interest paid (Given $a_{\overline{120}/0.01}$ = 69.6891).

22. Construct 5-yearly moving averages from the following data of the number of industrial failures in a country **[2]**

during 2003-2018:

Year	No. of failures	Year	No. of Failures
2003	23	2011	9
2004	26	2012	13
2005	28	2013	11
2006	32	2014	14
2007	20	2015	12
2008	12	2016	9
2009	12	2017	3
2010	10	2018	1

23. By using property of definite integrals, evaluate $\int_{-6}^{3} |x+3| dx$ **[2]**

24. If A = $\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$ and B = $\begin{bmatrix} -1 & 3 \\ 4 & 0 \end{bmatrix}$, verify that adj(AB) = (adj B)(adj A). **[2]**

OR

If $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$ then show that |3A| = 27|A|

25. Evaluate: (8×7) mod 6 **[2]**

Section C

26. A machine costing ₹ 30,000 is expected to have a useful life of 4 years and a final scrap value of ₹ 4000. Find the annual depreciation charge using the straight-line method. Prepare the depreciation schedule. **[3]**

27. Form the differential equation corresponding to $(x - a)^2 + (y - b)^2 = r^2$ by eliminating a and b. **[3]**

OR

Solve: $(x^2 + 1)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to the initial condition y(0) = 0.

28. The marginal revenue function of a commodity is given by MR = $11 - 3x + 4x^2$, find the revenue function. Also, find the demand function. **[3]**

29. The average number, in lakhs, of working days lost in strikes during each year of the period 2001-2010 was **[3]**

2001	2002	2003	2004	2005	2006	2007	2008	2009	2010
1.5	1.8	1.9	2.2	2.6	3.7	2.2	6.4	3.6	5.4

Calculate the three-yearly moving averages and draw the moving averages graph.

30. Consider the following hypothesis test: **[3]**

$H_0 : \mu = 18$

$H_a : \mu \neq 18$

A sample of 48 provided a sample mean $\bar{x} = 17$ and a sample standard deviation S = 4.5

i. Compute the value of the test statistic.
ii. Use the t-distribution table to compute a range for the p-value.
iii. At $\alpha = 0.05$, what is your conclusion?
iv. What is the rejection rule using the critical value? What is your conclusion?

31. A bag contains 8 red and 5 white balls. Two successive draws of all 3 balls are made at random from the bag without replacements. Find the probability that the first draw yields 3 white balls and second draw yields 3 red balls. **[3]**

OR

A pair of dice is thrown 7 times. If getting a total of 7 is considered a success, what is the probability of

i. no success?
ii. 6 successes?
iii. at least 6 successes?
iv. at most 6 successes?

Section D

32. Solve the linear programming problem graphically: **[5]**

Minimize Z = 3x + 5y subject to the constraints

$x + 3y \geq 3$, $x + y \geq 2$, $x \geq 0$, $y \geq 0$

OR

Solve the linear programming problems by graphical method:

Minimize Z = 2x + 4y

Subject to $x + y \geq 8$

$x + 4y \geq 12$

$x \geq 3$, $y \geq 2$

33. Suppose 220 misprints are distributed randomly throughout a book of 200 pages. Find the probability that a given page contains **[5]**

i. no misprints,
ii. one misprint,
iii. 2 misprints,
iv. 2 or more misprints.

(Given $e^{-1.1} = 0.33287$)

OR

Find the probability distribution of the number of doublets in 4 throws of a pair of dice. Also, find the mean and variance of this distribution.

34. In a 10 km race, A, B and C each running at uniform speed get the gold, silver and bronze medals respectively. If A beats B by 1 km and B beats C by 1 km, then find how many metres does A beat C. **[5]**

35. A machine costing ₹ 200000 has an effective life of 7 years and its scrap value is ₹ 30000. What amount should the company put into a sinking fund earning 5% per annum, so that it can replace the machine after its useful life? Assume that a new machine will cost ₹ 300000 after 7 years. **[5]**

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A cable network provider in a small town has 500 subscribers and he used to collect ₹ 300 per month from each subscriber. He proposes to increase the monthly charges and it is believed from past experience that for every increase of ₹ 1, one subscriber will discontinue the service.

i. If ₹ x is the monthly increase in subscription amount, then find the number of subscribers. (1)

ii. Find total revenue R (in ₹). (1)

iii. Find the number of subscribers which gives the maximum revenue. (2)

OR

Find the maximum revenue generated. (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Flexible payment arrangements, in which the borrower might pay higher sums of his or her choosing, are not the same as EMIs. Borrowers on EMI programmes are usually only allowed to make one set payment per month. Borrowers profit from an EMI since they know exactly how much money they will have to pay towards their loan each month, making personal financial planning easier. Lenders benefit from the loan interest, as it provides a consistent and predictable stream of income.

Example:

A loan of ₹400000 at the interest rate of 6.75% p.a. compounded monthly is to be amortized by equal payments at the end of each month for 10 years.

(Given $(1.005625)^{120} = 1.9603$, $(1.005625)^{60}$ - 1.4001)

i. Find the size of each monthly payment. (1)

ii. Find the principal outstanding at the beginning of 61st month. (1)

iii. Find the interest paid in 61st payment. (2)

OR

Find the principal contained in 61st payment. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

Two farmers Ramakishan and Gurucharan Singh cultivate only three varieties of rice namely Basmati, Permal and Naura. The sale (in rupees) of these varieties of rice by both the farmers in the month of September and October are given by the following matrices A and B.

September sales (in Rupees)

$$A = \begin{array}{c} \begin{bmatrix} 10,000 & 20,000 & 30,000 \\ 50,000 & 30,000 & 10,000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramakishan} \\ \text{Gurucharan} \end{array}$$

October sales (in Rupees)

$$B = \begin{array}{c} \begin{bmatrix} 5,000 & 10,000 & 6,000 \\ 20,000 & 10,000 & 10,000 \end{bmatrix} \end{array} \begin{array}{l} \text{Ramakishan} \\ \text{Gurucharan} \end{array}$$

Based on the information given above, answer the following questions:

i. The total sales in September and October for each farmer in each variety can be represented as _________. (1)

ii. What is the value of A_{23}? (1)

iii. If Ramkishan receives 2% profit on gross sales, compute his profit for each variety sold in October. (2)

OR

If Gurucharan receives 2% profit on gross sales, compute his profit for each variety sold in September. (2)

Solution

Section A

1.

(d) -4

Explanation:

The value of x for which the point (2, -1) (-3, 4) and (x, 5) are collinear is

$\because$ (2, -1) (-3, 4) and (x, 5) are collinear then.

Area of Δ = 0

$\frac{1}{2}\begin{vmatrix} 2 & -1 & 1 \\ -3 & 4 & 1 \\ x & 5 & 1 \end{vmatrix} = 0$

$\Rightarrow 2\begin{vmatrix} 4 & 1 \\ 5 & 1 \end{vmatrix} - (-1)\begin{vmatrix} -3 & 1 \\ x & 1 \end{vmatrix} + 1\begin{vmatrix} -3 & 4 \\ x & 5 \end{vmatrix} = 0$

$\Rightarrow$ 2(4 - 5) + (-3 - x) + 1(-15 - 4x) = 0

$\Rightarrow$ 2(-1) - 3 - x - 15 - 4x = 0

$\Rightarrow$ -2 - 18 - 5x = 0

$\Rightarrow$ -20 - 5x = 0

$\Rightarrow$ 5x = -20

$x = \frac{-20}{5}$

x = -4

2.

(c) $\mathrm{t} = \overline{\mathrm{x}} - \mu$

Explanation:

$\mathrm{t} = \overline{\mathrm{x}} - \mu$

3.

(b) ₹ 121.44

Explanation:

P = ₹ 2400, r = 5% p.a. or $\frac{5}{2}\%$ half yearly

Time = 2 half years

$\therefore$ C.I. = 2400 $\left[\left(1 + \frac{5}{200}\right)^2 - 1\right]$

= 2400[$(1.025)^2$ - 1]

= 2400[1.0506 - 1] = ₹ 121.44

4.

(d) (3, 4)

Explanation:

The values of Z = 5x + 7y at points (0, 0), (7, 0), (3, 4) and (0, 2) are 0, 35, 43 and 14 respectively.

Hence, maximum value of Z = 43 occurs at (3, 4).

5.

(d) 1 + a + b + c

Explanation:

Operating $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} 1+a & b & c \\ a & 1+b & c \\ a & b & 1+c \end{vmatrix} = \begin{vmatrix} 1+a+b+c & b & c \\ 1+a+b+c & 1+b & c \\ 1+a+b+c & b & 1+c \end{vmatrix}$$ (Take 1 + a + b + c common from C_1)

$= (1 + a + b + c)\begin{vmatrix} 1 & b & c \\ 1 & 1+b & c \\ 1 & b & 1+c \end{vmatrix}$

$= (1 + a + b + c)\begin{vmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

(Operate $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$)

= (1 + a + b + c).1(1 - 0) (Expand by C_1)

= 1 + a + b + c

6.

(b) $\frac{1}{e}$

Explanation:

Given P(X = 1) = 2 P(X = 2)

$\Rightarrow \frac{\lambda^1 e^{-\lambda}}{1!} = 2 \times \frac{\lambda^2 . e^{-\lambda}}{2!}$

$\Rightarrow \lambda = \lambda^2 \Rightarrow \lambda = 0, 1 \Rightarrow \lambda = 1$

Now, P(X = 0) $= \frac{\lambda^0 . e^{-\lambda}}{0!} = e^{-1} = \frac{1}{e}$

7.

(c) $\frac{1}{8}, \frac{3}{4}$

Explanation:

Since the sum of probabilities in a probability distribution is 1, therefore,we have,

P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = 1

k + 3k + 3 k + k = 1

8k = 1

k = $\frac{1}{8}$

Mean = 0 × k + 1 × 3k + 2 × 3k + 3 × k

= 12k = 12 × $\frac{1}{8}$=$\frac{3}{2}$

Variance = $\sum p_i x_i^2 - (mean)^2$

= 0 × k + 1^2 × 3k + 2^2 × 3k + 3^2 × $k - (\frac{3}{2})^2$

= 24k - $\frac{9}{4}$ = 24×$\frac{1}{8} - \frac{9}{4} = 3 - \frac{9}{4} = \frac{3}{4}$

8. **(a)** x

Explanation:

$\frac{dy}{dx} + \frac{y}{x}$ = 3x, which is linear in y. I.F. = $e^{\int \frac{1}{x} dx}$ = $e^{\log x dx}$ = x

9. **(a)** 8 km

Explanation:

The speed of boat in still water = 15 km/hr

Speed of water current = 5 km/hr

∴ Speed in down stream = 15 + 5 = 20 km/hr

Time given = 24 min = $\frac{24}{60}$ hr = $\frac{2}{5}$ hr

∴ Distance travelled = speed × times

= 20 × $\frac{2}{5}$ = 8 km

10. **(a)** 6

Explanation:

18 → 1 × 18, 2 × 9, 3 × 6, 6 × 3, 9 × 2, 18 × 1

11.

(b) 6

Explanation:

6

12.

(c) $\frac{1}{x^2}$

Explanation:

If 0 < x < 1, then x^2 < x but $\frac{1}{x^2} > \frac{1}{x}$ ($\because$ x > 0)

Also, $\frac{1}{x}$ > 1, so $\frac{1}{x}$ > x

Thus, $\frac{1}{x^2} > \frac{1}{x}$ > x > x^2

$\therefore \frac{1}{x^2}$ is greatest.

13. **(a)** 45 : 41

Explanation:

A : B = 100 : 90

A : C = 100 : 82

$\therefore \frac{B}{C} = (\frac{B}{A} \times \frac{A}{C})$ = $\frac{90}{100} \times \frac{100}{82}$ = 45 : 41

14.

(b) Only first quadrant

Explanation:

Only first quadrant

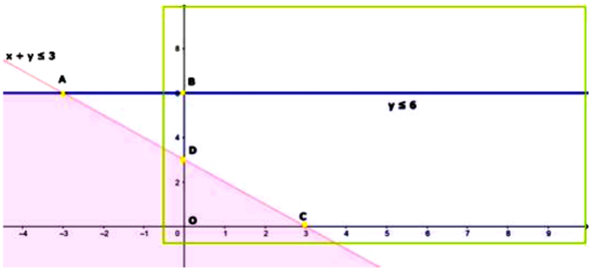
15. **(a)** unbounded in first quadrant

Explanation:

Given inequations are

x, y $\geq$ 0, y $\leq$ 6, x + y $\leq$ 3

If we graph the inequalities y $\leq$ 6 and x + y $\leq$ 3 on the graph, we get the graph,



Here we can clearly see that, the region for the inequality is above the points A, B, C and D. But considering the constraints x, y $\geq$ 0, this clearly shows that the region can only be in the 1^{st} quadrant.

So from the green color highlighted area, we can say that, the region of the inequalities is unbound in the first quadrant.

16. **(a)** 0.34

Explanation:

0.34

17.

(c) $-\frac{1}{\log_e 2}$

Explanation:

We have to find: $I = \int \frac{2^{\frac{1}{x}}}{x^2} dx$

Put $\frac{1}{x} = t$

$\Rightarrow \frac{-1}{x^2} dx = dt \Rightarrow \frac{1}{x^2} dx = -dt$

$\therefore\ I = \int 2^t(-dt)$

$I = \frac{-2^t}{\log_e 2} + c$

$\Rightarrow I = \frac{-2^{\frac{1}{x}}}{\log_e 2} + c$

$\Rightarrow k = \frac{-1}{\log_e 2}$

18. **(b)** $\frac{a+c+d+e}{4}$

Explanation:

$\frac{a+c+d+e}{4}$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

The given matrices are A = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

Then, A + B = $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$= \begin{bmatrix} 2+1 & 4+3 \\ 3-2 & 2+5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

20. **(b)** Both A and R are true but R is not the correct explanation of A.

Explanation:

The equation of the given curve is $\frac{1}{x^2-2x+3}$ ...(i)

The slope of the tangent to the given curve at any point (x, y) is given by

$\frac{dy}{dx} = \frac{-1}{(x^2-2x+3)^2} \frac{d}{dx}$ (x^2 - 2x + 3)

= $\frac{-(2x-2)}{(x^2-2x+3)^2} = \frac{-2(x-1)}{(x^2-2x+3)^2}$

For all tangents having slope 0, we must have $\frac{dy}{dx}$ = 0

$\Rightarrow \frac{-2(x-1)}{(x^2-2x+3)^2}$ = 0

$\Rightarrow$ -2(x - 1) = 0 $\Rightarrow$ x = 1

From Eq. (i), we get

y = $\frac{1}{1^2-2\times1+3} = \frac{1}{2}$

$\therefore$ The equation of tangent to the given curve at point $\left(1, \frac{1}{2}\right)$ having slope = 0 is y - $\frac{1}{2}$ = 0(x - 1) $\Rightarrow$ y = $\frac{1}{2}$

Hence, the equation of the required line is y = $\frac{1}{2}$

Hence, both Assertion and Reason are true.

Section B

21. i. Given r = 6% p.a.

p = 4 quarters.

So, effective rate (per rupee) = $\left(1 + \frac{6}{400}\right)^4 - 1$ = $(1.015)^4$ - 1

= 1.06136 - 1 = 0.06136 = 0.0614

Hence, effective rate = 0.0614 $\times$ 100% = 6.14%

ii. Given r = 6% p.a.

So, i = $\frac{6}{100}$ = 0.06

When compounded continuously, then

effective rate (per rupee) = e^i - 1 = $e^{0.06}$ - 1

= 1.06183 - 1 = 0.06183 = 0.0618

Hence, effective rate = 0.0618 $\times$ 100% = 6.18%

OR

P = ₹ 150000, i = $\frac{12}{1200}$= = 0.01, n = 10 $\times$ 12 = 120

i. EMI = $\frac{P}{a_{\bar{n}/i}} = \frac{150000}{a_{120/0.01}} = \frac{150000}{69.6891}$ = ₹ 2152.43.

ii. Total interest paid = n $\times$ EMI - P

= 120 $\times$ 2152.42 - 150000

= ₹108290.40

22. Computation of moving averages

Year	**No. of failures**	**5-yearly moving totals**	**5-yearly moving averages**
2003	23	-	-
2004	26	-	-
2005	28	129	25.8
2006	32	118	23.6
2007	20	104	20.8
2008	12	86	17.2
2009	12	63	12.6
2010	10	56	11.2
2011	9	55	11.0
2012	13	57	11.4
2013	11	59	11.8
2014	14	59	11.8
2015	12	49	9.8
2016	9	39	7.8
2017	3	-	-
2018	1	-	-

23. $\int\limits_{-6}^{3}|x+3|dx = \int\limits_{-6}^{-3}|x+3|dx + \int\limits_{-3}^{3}|x+3|dx$

$= \int\limits_{-6}^{-3}-(x+3)dx + \int\limits_{-3}^{3}(x+3)dx = \left[-\frac{x^2}{2}-3x\right]_{-6}^{-3} + \left[\frac{x^2}{2}+3x\right]_{-3}^{3}$

$= \left(-\frac{9}{2}+9\right) - (-18+18) + \left(\frac{9}{2}+9\right) - \left(\frac{9}{2}-9\right) = \frac{45}{2}$

24. Given A = $\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}$ $\Rightarrow$ adj A = $\begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$

and B = $\begin{bmatrix} -1 & 3 \\ 4 & 0 \end{bmatrix}$ $\Rightarrow$ adj B = $\begin{bmatrix} 0 & -3 \\ -4 & -1 \end{bmatrix}$

AB = $\begin{bmatrix} 2 & -3 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -1 & 3 \\ 4 & 0 \end{bmatrix}$ = $\begin{bmatrix} -14 & 6 \\ -1 & 15 \end{bmatrix}$

$\Rightarrow$ adj(AB) = $\begin{bmatrix} 15 & -6 \\ 1 & -14 \end{bmatrix}$.

Also, (adj B) (adj A) = $\begin{bmatrix} 0 & -3 \\ -4 & -1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -5 & 2 \end{bmatrix}$ = $\begin{bmatrix} 15 & -6 \\ 1 & -14 \end{bmatrix}$

$\therefore$ adj(AB) = (adj B)(adj A)

OR

Given: $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then $3A = 3\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

L.H.S. = $|3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$

= 3(36 - 0) = 3 $\times$ 36 = 108

R.H.S. = 27|A| $= 27\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

= 27[1(4 - 0)] = 27 $\times$ 4 = 108

Since L.H.S. = R.H.S.

Hence, proved.

25. To find (8 $\times$ 7) mod 6, let us divide 8 $\times$ 7 i.e. 56 by 6

$$\begin{array}{r|l} 6\,)\;56 & 9 \\ 54 & \\ \hline 2 \longrightarrow \text{Remainder} & \end{array}$$

So, (8 $\times$ 7) mod 6 = 2

Section C

26. We are given that

C = 30,000; n = 4; S = 4000

Annual depreciation = $\frac{C-S}{n}$

$= \frac{30000-4000}{4}$

= 6500

Depreciation schedule

Year	**Annual depreciation**	**Accumulated depreciation**	**Book Value**
0	0	0	30,000
1	6500	6500	23,500
2	6500	13000	17,000
3	6500	19,500	10,500
4	6500	26,000	4000

27. The equation of the family of curves is $(x - a)^2 + (y - b)^2 = r^2$ ...(i)

where a and b are parameters.

This equation contains two parameters, so we shall get a second order differential equation.

Differentiating equation (i) with respect to x, we get

$2(x - a) + 2(y - b) \frac{dy}{dx} = 0$

$(x - a) + (y - b)\frac{dy}{dx} = 0$ ...(ii)

Again differentiating w.r.t x, we get,

$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 + (y - b)\frac{d^2y}{dx^2} = 0$

$\Rightarrow (y - b) = -\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ ...(iii)

From (ii) and (iii), we have,

$(x - a) - \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \frac{dy}{dx} = 0$

$\Rightarrow (x - a) = \frac{\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3}{\frac{d^2y}{dx^2}}$ ...(iv)

From (i), (iii) and (iv), we get

$\frac{\left[\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^3\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2} + \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$

$\Rightarrow \frac{\left[\left(\frac{dy}{dx}\right)^2+2\left(\frac{dy}{dx}\right)^4+\left(\frac{dy}{dx}\right)^6\right]+\left[1+2\left(\frac{dy}{dx}\right)^2+\left(\frac{dy}{dx}\right)^4\right]}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)^4 + \left(\frac{dy}{dx}\right)^6 + 1 + 2\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^4 = r^2\left(\frac{d^2y}{dx^2}\right)^2$

$\Rightarrow 1 + 3\left(\frac{dy}{dx}\right)^2 + 3\left(\frac{dy}{dx}\right)^4 + \left(\frac{dy}{dx}\right)^6 = r^2\left(\frac{d^2y}{dx^2}\right)^2$

$\Rightarrow \left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = r^2\left(\frac{d^2y}{dx^2}\right)^2$

It is the required differential equation.

OR

The given differential equation can be written as

$\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$ ...(i)

This is a linear differential equation of the form $\frac{dy}{dx}$ + Py = Q, where

P = $\frac{2x}{1+x^2}$ and Q = $\frac{4x^2}{1+x^2}$

$\therefore$ I.F. = $e^{\int Pdx} = e^{\int \frac{2x}{(1+x^2)}dx} = e^{\log(1+x^2)}$ = 1 + x^2

Multiplying both sides of (i) by I.F. = (1 + x^2), we get

(1 + x^2)$\frac{dy}{dx}$ + 2xy = $4x^2$

Integrating both sides with respect to x, we get

y(1 + x^2) = $\int$ $4x^2$ dx + C [Using: y(I.F.) = $\int$ Q (I.F.) dx + C]

$\Rightarrow$ y (1 + x^2) = $\frac{4x^3}{3}$ + C ...(ii)

It is given that y = 0, when x = 0. Putting x = 0 and y = 0 in (i), we get

0 = 0 + C $\Rightarrow$ C = 0

Substituting C = 0 in (ii), we get y = $\frac{4x^3}{3(1+x^2)}$, which is the required solution.

28. Let R(x) be the revenue function of x units of the product and MR be the marginal revenue function, then

MR = 11 - 3x + $4x^2$.

As MR = $\frac{d}{dx}$ (R(x)), so $\frac{d}{dx}$ (R(x)) = 11 - 3x + $4x^2$

$\therefore$ R(x) = $\int$ (11 - 3x + $4x^2$) dx

= 11x - $3 \cdot \frac{x^2}{2} + 4 \cdot \frac{x^3}{3}$ + k, where k is constant of integration.

When x = 0, R(x) = 0

$\Rightarrow$ 0 = 11 $\times$ 0 - $\frac{3}{2}$ $\times$ 0 + $\frac{4}{3}$ $\times$ 0 + k $\Rightarrow$ k = 0.

$\therefore$ R(x) = 11x - $\frac{3}{2}x^2$ + $\frac{4}{3}x^3$.

If p is the price per unit when x units of the product are sold, then

R(x) = p·x

$\Rightarrow$ px = 11x - $\frac{3}{2}x^2$ + $\frac{4}{3}x^3$

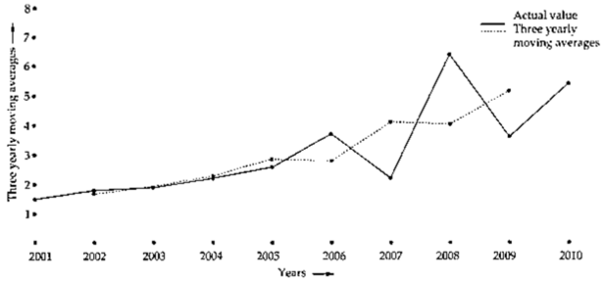
$\Rightarrow$ p = 11 - $\frac{3}{2}$x + $\frac{4}{3}x^2$, which is the corresponding demand function.

29. In order to calculate three-yearly moving averages, we first compute three-yearly moving totals and place each total against the middle year of the three-year span from which the moving totals are calculated. These moving totals are given in the third column of the following table. From these three yearly moving totals, we calculate three-yearly moving averages by dividing each moving total by 3 as shown in the following table.

Year	**Working days lost in strikes (in lakhs)**	**Three yearly moving totals**	**Three yearly moving averages**
2001	1.5	-	-
2002	1.8	5.2	1.73
2003	1.9	5.9	1.96
2004	2.2	6.7	2.23
2005	2.6	8.5	2.83
2006	3.7	8.5	2.83
2007	2.2	12.3	4.1

2008	6.4	12.2	4.06
2009	3.6	15.4	5.13
2010	5.4	-	-

The graph of these moving averages is shown in Fig.



30. Given μ_0 = 18, n = 48, $\bar{x}$ = 17, S = 4.5

i. t = $\frac{\bar{x}-\mu_0}{\frac{S}{\sqrt{n}}} = \frac{17-18}{\frac{4.5}{\sqrt{48}}}$

$= \frac{-1\times\sqrt{48}}{4.5}$ = -1.54

$\therefore$ t = -1.54

and degrees of freedom = 48 - 1 = 47.

ii. $\because$ t = -1.54 < 0

So, p-value of -1.54 = 2 $\times$ Area under the t-distribution curve to the left of t

= 2 $\times$ Area under the t-distribution curve to the right of t

From the t-distribution table, we find that t = 1.54 lies between 1.300 and 1.678 for which area lies between 0.05 and 0.10, so, p-values lies between 2 $\times$ 0.05 and 2 $\times$ 0.10 i.e. between 0.10 and 0.20.

So, 0.10 < p-value < 0.20

iii. $\because$ p-value > 0.05

So, do not reject H_0.

iv. Reject H_0 if t $\leq t_{\frac{\alpha}{2}}$ or t $\geq t_{\frac{\alpha}{2}}$.

Here, t = -1.54 and $t_{\frac{\alpha}{2}}$ = $t_{0.025}$

From the table, $t_{0.025}$ = 2.012 with df = 47

$\because$ -1.54 > -2.012

So, do not reject H_0

31. Let E : Event that 3 balls in the first draw are all white.

F : Event that 3 balls in the second draw are all red.

Now, 3 balls can be drawn out of 13 in $^{13}C_3$ ways and 3 white balls can be drawn out of 5 in $^{5}C_3$ ways

P(E) = $\frac{^5C_3}{^{13}C_3} = \frac{5!}{3!\times2!} \times \frac{3!\times10!}{13!} = \frac{5}{143}$

Since, 3 balls are not replaced before the second draw, we are left with 8 red and 2 white balls.

Now, 3 balls can be drawn in $^{10}C_3$ ways and 3 red balls can be drawn in $^{8}C_3$ ways.

$P\left(\frac{F}{E}\right) = \frac{^8C_3}{^{10}C_3}$

$= \frac{8!}{3!\times5!} \times \frac{3!\times7!}{10!} = \frac{7}{15}$

$\therefore P(E\cap F) = P(E).P\left(\frac{F}{E}\right) = \frac{5}{143} \times \frac{7}{15}$

= $\frac{7}{429}$.

OR

Let p denote the probability of getting a total of 7 in a single throw of a pair of dice.

Then, $p = \frac{6}{36} = \frac{1}{6}$ [$\because$ The sum can be 7 in any one of the ways : (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)]

$\therefore$ q = 1 - p $= 1 - \frac{1}{6} = \frac{5}{6}$

Let X denote the number of successes in 7 throws of a pair of dice. The X is a binomial variate with parameters n = 7 and p = $\frac{1}{6}$ such that

P(X = r) $= {^7C_r}\left(\frac{1}{6}\right)^r\left(\frac{5}{6}\right)^{7-r}$, r 0, 1, 2, ..., 7 ...(i)

i. Probability of no success = P(X = 0) = ${}^7C_0\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{7-0}=\left(\frac{5}{6}\right)^7$ [Using (i)]

ii. Probability of 6 success = P(X = 0) =${}^7C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^{7-6}=35\left(\frac{1}{6}\right)^7$ [Using (i)]

iii. Probability of at least 6 successes = P (X $\geq$ 6)

= P (X = 6) + P (X = 7)

$={}^7C_6\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)^{7-6}+{}^7C_7\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right)^0$ [Using (i)]

$=7\left(\frac{1}{6}\right)^6\left(\frac{5}{6}\right)+\left(\frac{1}{6}\right)^7=\left(\frac{1}{6}\right)^6\left(\frac{35}{6}+\frac{1}{6}\right)=\left(\frac{1}{6}\right)^5$

iv. Probability of at most 6 successes = P (X $\leq$ 6)

= 1 - P (X > 6)

= 1 - P (X = 7)

$=1-{}^7C_7\left(\frac{1}{6}\right)^7\left(\frac{5}{6}\right)^0=1-\left(\frac{1}{6}\right)^7$ [Using (i)]

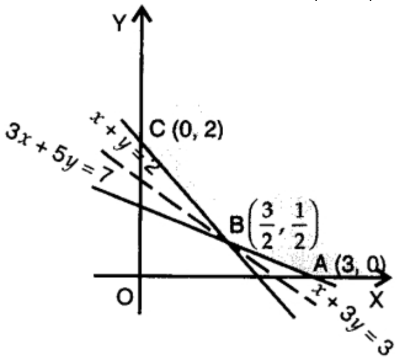
Section D

32. Minimize Z = 3x + 5y subject to the constraints

x + 3y $\geq$ 3, x + y $\geq$ 2 , x $\geq$ 0, y $\geq$ 0

The feasible region is shown shaded in the adjoining figure. It is convex and unbounded.

Corner points are A (3, 0) B $\left(\frac{3}{2},\frac{1}{2}\right)$ and C(0, 2).



The values of Z at the comer points A, Band Care 9, 7 and 10 respectively.

Among the values of Z, minimum value of Z is 7

As the feasible region is unbounded, we draw the graph of the half plane 3x + 5y < 7 and note that there is no point common with the feasible region, therefore, Z has minimum value and the minimum value = 7 and it occurs at $x=\frac{3}{2},y=\frac{1}{2}$.

OR

Given,

Objective function is: Z = 2x + 4y

Constraints are:

x + y $\geq$ 8

x + 4y $\geq$ 12

x $\geq$ 3

y $\geq$ 2

First convert the given inequations into corresponding equations and plot them:

x + y $\geq$ 8 $\Rightarrow$ x + y = 8 (corresponding equation)

Two coordinates required to plot the equation are obtained as:

Put, x = 0 $\Rightarrow$ y = 8(0, 8) ...first coordinate.

Put, y = 0 $\Rightarrow$x = 8(8, 0) ...second coordinate.

Join them to get the line.

As we know, Linear inequation will be a region in the plane and we observe that the equation divides the XY plane into 2 halves only, so we need to check which region represents the given inequation.

If the given line does not pass through origin then just put (0, 0) to check whether inequation is satisfied or not. If it satisfies the inequation origin side is required else the other side is the solution.

Similarly, we repeat the steps for other inequation also and find the common region.

As, x + 4y $\geq$ 12 $\Rightarrow$ x + 4y = 12

Put x = 0 $\Rightarrow$ y = 3 coordinate ...(0, 3)

Put y = 0 $\Rightarrow$ x = 12 coordinate ...(12, 0) y = 2 (line parallel to x-axis passing through (0, 2))

x = 3 (line parallel to x-axis passing through (3, 0))

Hence, we obtain a plot as shown in figure.

The shaded region in the above figure represents the region of feasible solution.

Now to maximize our objective function, we need to find the coordinates of hte corner points of the shaded region.

we can determine the coordinates graphically our by solving equations. But choose only those equations to solve which gives one of the corner coordinates of the feasible region.

Solving x + y = 8 and x = 3 gives (3, 5)

Similarly, solve other combinations by observing graph to get other coordinates.

From the figure we have obtained coordinates of corner as:

(3, 5) and (6, 2)

Now we have coordinates of the corner points so we will put them one by one to our objective function and will find at which point it is maximum.

$\because$ Z = 2x + 4y

$\therefore$ Z at (3, 5) = 2 × (3) + 4 × (5) = 26

Z at (6, 2) = 2 × (6) + 4 × (8) = 20

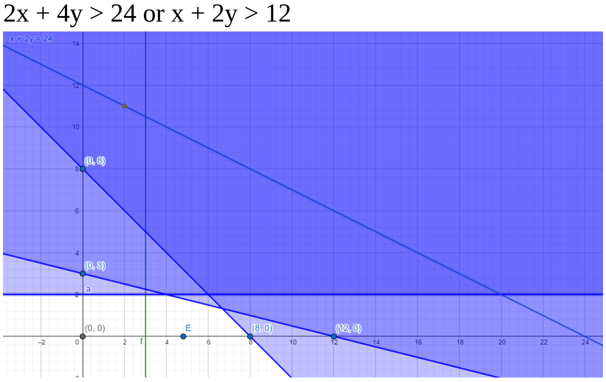
Note: As the region is unbounded as we can't say blindly that Z = 24 is maximum because there might be other points in the feasible region that may Make Z even greater.

So we need to check whether Z is maximum or not or Z greater than 24 or not.

For this we define inequation using the optimal function if the solution region of the inequation does not coincide with the feasible region, it means it has a maxima

Inequation:

2x + 4y > 24 or x + 2y > 12



Now we again plot the graph with the constraints and the above inequation

Clearly, x + 2y > 24 has solutions in feasible region.

This proves that the values of Z greater than 24 are possible.

$\therefore$ The optimal value of Z is not possible.

33. Let p be the probability of selecting a page out of 200 pages. Then,

$p = \frac{1}{200} = 0.005$

Since p is small, we use the Poisson's distribution

Here, n = Total number of misprints = 220

Average number of misprints in a page= n p

$\Rightarrow$ m = np $\Rightarrow$ m = 220 $\times$ 0.005 = 1.1

Let X denote the number of misprints in a page. Then, X follows Poisson's distribution such that

P(X = r) = $\frac{m^r}{r!}$ e^{-m}, r = 0, 1, 2, 3, ...

$= \frac{(1.1)^r}{r!}$ $e^{1.1}$, r = 0, 1, 2, ...

i. Required probability = P (X = 0)

= $e^{-1.1}$ = 0.33287

ii. Required probability = P (X = 1)

$= \frac{1.1}{1!}$ $e^{-1.1}$ = 1.1 $\times$ 0.33287 = 0.366157

iii. Required probability = P (X = 2)

$= \frac{m^2 e^{-m}}{2!}$

$= \frac{(1.1)^2}{2} \times$ $e^{-1.1}$

$= \frac{1.21 \times 0.33287}{2}$

= 0.20138

iv. Required probability = P (X > 2)

= 1 - P(X < 2)

= 1 - [P(X = 0) + P(X = 1)]

= 1 - [0.33287 + 0.366157] [Using (i) and (ii)]

= 0.300973

OR

We have to find the probability distribution of the number of doublets in four throws of a pair of dice. Also,We need to find the mean and variance of this distribution.

We know that, the probability of getting doublet

$= \frac{6}{36} = \frac{1}{6}$

[doublets in a pair of dice are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) and (6, 6)]

i.e. $p = \frac{1}{6}$

Probability of not getting a doublet

= 1- p

$= 1 - \frac{1}{6} = \frac{5}{6}$

i.e. $q = \frac{5}{6}$

Here, we have n = 4, $p = \frac{1}{6}, q = \frac{5}{6}$

Let X denotes the number of doublets, then X can take values 0, 1, 2, 3 and 4.

We know that, by binomial distribution

$P(X) = {}^nC_r p^r q^{n-r}$

$\therefore P(X = 0) = {}^4C_0 \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4$

$= 1 \times 1 \times \left(\frac{5}{6}\right)^4 = \frac{625}{1296}$

$P(X = 1) = {}^4C_1 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3$

$= 4 \times \frac{1}{6} \times \frac{125}{215} = \frac{500}{1296}$

$P(X = 2) = {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2$

$= \frac{4!}{2!2!} \times \frac{1}{36} \times \frac{25}{36} = \frac{6 \times 25}{36 \times 36} = \frac{150}{1296}$

$P(X = 3) = {}^4C_3 \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1$

$= 4 \times \frac{1}{216} \times \frac{5}{6} = \frac{20}{1296}$

$P(X = 4) = {}^4C_4 (\frac{1}{6})^4 (\frac{5}{6})^0$

$= 1 \times \left(\frac{1}{6}\right)^4 \times 1 = \frac{1}{1296}$

$\therefore$ The required probability distribution is as follows

X	0	1	2	3	4
P(X)	$\frac{625}{1296}$	$\frac{500}{1296}$	$\frac{150}{1296}$	$\frac{20}{1296}$	$\frac{1}{1296}$

$\therefore$ Mean $= \sum X_i P(X_i)$

$= 0 \times \frac{625}{1296} + 1 \times \frac{500}{1296} + 2 \times \frac{150}{1296}$

$+3 \times \frac{20}{1296} + 4 \times \frac{1}{1296}$

$= 0 + \frac{500}{1296} + \frac{300}{1296} + \frac{60}{1296} + \frac{4}{1296} = \frac{864}{1296} = \frac{2}{3}$

and variance= $\sum X_i^2 P(X_i) - (Mean)^2$

$= \left[0 \times \frac{625}{1296} + (1)^2 \times \frac{500}{1296} + (2)^2 \times \frac{150}{1296} + (3)^2 \times \frac{20}{1296} + (4)^2 \times \frac{1}{1296}\right] - \left(\frac{2}{3}\right)^2$

$= \left[0 + \frac{500}{1296} + \frac{600}{1296} + \frac{180}{1296} + \frac{16}{1296}\right] - \left(\frac{2}{3}\right)^2$

$= \left[\frac{1296}{1296}\right] - \left(\frac{2}{3}\right)^2 = 1 - \left(\frac{4}{9}\right) = \left(\frac{5}{9}\right)$

34. 1900 m

35. Cost of new machine = ₹ 300000

Scrap value of old machine = ₹ 30000

Hence, the money required for new machine after 7 years

= ₹ 300000 - ₹ 30000 = ₹ 270000

So, we have A = ₹ 270000, i = $\frac{5}{100}$ = 0.05, n = 7

Using formula, A = R$\left[\frac{(1+i)^n - 1}{i}\right]$, we get

270000 = R$\left[\frac{(1.05)^7 - 1}{0.05}\right]$

$\Rightarrow$ R = $\frac{270000 \times 0.05}{(1.05)^7 - 1}$ [Let x = $(1.05)^7 \Rightarrow$ x = 7 log 1.05 = 7 $\times$ 0.0212 = 0.1484 $\Rightarrow$ x = antilog 0.1484 $\Rightarrow$ x = 1.407]

$\Rightarrow$ R = $\frac{13500}{1.407 - 1} = \frac{13500}{0.407}$

$\Rightarrow$ R = 33169.53

Hence, the company should deposit ₹ 33169.53 at the end of each year for 7 years.

Section E

36. i. Since with increase of ₹ 1 in subscription, one subscriber discontinues the service and there is an increase of ₹ x in subscription, so the number of subscribers left = 500 - x.

ii. New monthly subscription charge (in ₹) = (300 + x)

$\Rightarrow$ total revenue (in ₹) R = (300 + x) (500 - x).

iii. $\frac{dR}{dx}$ = (500 - x) $\times$ 1 + (-1)(300 + x) = 200 - 2x and $\frac{d^2R}{dx^2}$ = -2.

Now, $\frac{dR}{dx}$ = 0 $\Rightarrow$ 200 - 2x = 0 $\Rightarrow$ x = 100.

At x = 100, $\frac{d^2R}{dx^2}$ = -2 < 0 $\Rightarrow$ R is maximum.

Thus, the revenue is maximum when the subscription is increased by ₹ 100.

The number of subscribers left = 500 - 100 = 400.

OR

New charges per subscriber = ₹ (300 + 100) = ₹ 400 and the number of subscribers left = 400.

Maximum revenue generated = ₹ (400 $\times$ 400) = ₹ 160000.

37. i. ₹ 4593

ii. ₹ 233336.89

iii. ₹ 1312.52

OR

₹ 3280.48

38. i. A + B

ii. 10000

iii. ₹100, ₹ 200 and ₹120

OR

₹1000, ₹600, ₹200